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Quantum Symmetries in the Maxwell–Chern–Simons Theory Coupled to Scalar Fields

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The Maxwell-Chern-Simons gauge theory coupled to a complex scalar field is quantized in the Becchi-Rouet-Stora-Tyutin (BRST) path integral formalism. On the basis of the symmetries of a constrained canonical (Hamiltonian) system, we get the quantal conserved angular momentum of the system under the global symmetry transformation. It is shown that fractional spin also appears at the quantum level. The canonical Ward identities for this system are derived under local gauge transformation.

KEY WORDS: constrained Hamiltonian systems; path integral quantization; symmetry and conservation laws; fractional spin.

1. INTRODUCTION

Fractional spin and statistics have attracted much attention because of their possible relevance to condensed matter phenomena, especially to the fractional quantum Hall effect (Semenoff and Sodano, 1986) and high- T_c superconductivity (Kalmeyer, and Laughlin, 1987). Fractional spin may appear in gauge theories with the Chern–Simons (CS) term (Semenoff, 1988). Theoretical understanding of them has been gained in the context of both quantum mechanics (Wilczek, 1982) and quantum field theory (Semenoff, 1988). So far, development in the direction of field theory has not progressed as far as that of quantum mechanics. In the study at the field-theoretical level, the Abelian CS theory minimally coupled to the matter fields is usually considered as the base system. Recently, some models of gauge-invariant theory have been suggested and investigated on the basis of the Maxwell CS theory in canonical approach (Gordon and Pasquale, 1989). The angular momentum for anyons based on the canonical approach is always obtained through the symmetric energy–momentum tensor, not Noether's law. Some authors have put forward that the expressions obtained by these two prescriptions may not be identical, although both generate appropriate transformations (Chakraborty and

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Majumdar, 1996, 1999). Second, in the canonical approach the operator ordering problem becomes quite severe if the analysis is carried out in the gauge fixed scheme, rather than in the gauge independent scheme (Majumdar and Chakraborty, 1996). Third, other symmetry properties of CS theory also need further study.

The system considered here is the Maxwell CS theory coupled to a complex scalar field. This paper is organized as follows. In section 2 the BRST path integral scheme is formulated, since the system has BRST symmetry. There is no operator ordering problem in the study of the angular momentum. In section 3 it is shown that fractional spin also appears through quantum Noether's prescription for this system. Finally the gauge generator for this system is constructed and the canonical Ward identities are deduced.

2. BRST PATH INTEGRAL QUANTIZATION

The Lagrangian density of the (2 + 1)-dimensional Abelian CS term coupled to the scalar field is given by Gordon and Pasquale, 1989

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}\varepsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda + (D_\mu\varphi)^*(D^\mu\varphi) + m^2\varphi^*\varphi \tag{1}$$

where $D_\mu = \partial_\mu - iA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The canonical momenta conjugate to the fields A_μ, φ, and φ^* are

$$\pi^i = \frac{\partial\mathcal{L}}{\partial\dot{A}_i} = F^{i0} + \frac{\kappa}{2}\varepsilon^{ij}A_j \tag{2a}$$

$$\pi^0 = \frac{\partial\mathcal{L}}{\partial\dot{A}_0} = 0 \tag{2b}$$

$$\pi_\varphi = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = (D_0\varphi)^*, \qquad \pi_\varphi^* = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}^*} = D_0\varphi \tag{2c}$$

respectively. The primary constraint of the system is given by

$$\Lambda^0 = \pi^0 \approx 0 \tag{3}$$

where symbol "≈" means weakly equality in Dirac sense. The canonical Hamiltonian density $\mathcal{H}_c$ is

$$\mathcal{H}_c = \pi^\mu\dot{A}_\mu + \pi_\varphi\dot{\varphi} + \pi_\varphi^*\dot{\varphi}^* - \mathcal{L} = \mathcal{H}_0 + A_0\left[J_0 - \left(\partial_i\pi^i + \frac{\kappa}{4}\varepsilon^{ij}F_{ij}\right)\right] \tag{4a}$$

with

$$\mathcal{H}_0 = \pi_\varphi\pi_\varphi^* - (D_i\varphi)^*(D^i\varphi) - m^2\varphi^*\varphi - \frac{1}{2}\pi^i\pi_i + \frac{\kappa}{2}\varepsilon^{ij}\pi_iA_j - \frac{\kappa^2}{8}A^iA_i + \frac{1}{4}F_{ij}F^{ij} \tag{4b}$$

and $J_0 = i(\pi_\varphi \varphi - \varphi^* \pi_\varphi^*)$. The total Hamiltonian is given by

$$H_{\rm T} = \int d^2x(\mathcal{H}_{\rm c} + \lambda_0 \Lambda^0) = \int d^2x \left\{ \mathcal{H}_0 + A_0 \left[J_0 - \left(\partial_i \pi^i + \frac{\kappa}{4} \varepsilon^{ij} F_{ij} \right) \right] + \lambda_0 \Lambda^0 \right\} \tag{5}$$

The consistency of the primary constraint, $\{\Lambda^0, H_{\rm T}\} \approx 0$, leads to the secondary constraint

$$\Lambda^1 = J_0 - \left(\partial_i \pi^i + \frac{\kappa}{4} \varepsilon^{ij} F_{ij} \right) \approx 0 \tag{6}$$

The consistency of the secondary constraint gives no new constraints. It is easy to see that the constraints (Λ^0, Λ^1) are first class. The extended Hamiltonian can be written as

$$H_{\rm E} = \int d^2x(\mathcal{H}_0 + \lambda_0 \Lambda^0 + A_0 \Lambda^1) = \int d^2x(\mathcal{H}_0 + \lambda_0 \Lambda^0 + \lambda_i \Lambda^1) \tag{7}$$

where λ_0 and λ_1 are Lagrange multipliers.

In the BRST quantization (Henneaux, 1985) scheme, the Lagrangian multipliers are treated as the dynamical variables of the system, hence the associated canonical momenta must be equal to zero in order that the physical content of the system is unchanged. From (7) it is clear that $\lambda_1 = A_0$, and one has $\pi^1 = \pi^0 \approx 0$, where π^1 is a canonical momentum conjugate to the Lagrange multiplier field λ_1. The original phase space $u_A(A_i, \varphi, \varphi^*; \pi^i, \pi_\varphi, \pi_\varphi^*)$ is replaced by $u_\Delta(u_A, \lambda_1 = A_0, \pi^1 = \pi^0)$ after the Lagrangian multiplier λ_1 is treated as the dynamical variable of the system. Now, the constraints of the system can be denoted by $G_a = (G_1, G_2) = (\Lambda^0, \Lambda^1)$. We associate with each constraint G_a a canonically conjugate pair of anticommuting ghosts $(\eta^a, \mathcal{P}_a)$ which can be denoted by

$$\eta = (-i\mathcal{P}, C), \quad \mathcal{P} = (i\bar{C}, \bar{\mathcal{P}}) \tag{8a}$$

The Poisson brackets of the ghosts $(\eta^a, \mathcal{P}_a)$ satisfy that

$$\{\mathcal{P}(x), \bar{C}(y)\} = -\delta(\vec{x} - \vec{y}), \quad \{\bar{\mathcal{P}}(x), C(y)\} = -\delta(\vec{x} - \vec{y}) \tag{8b}$$

and others are qual to zero. The extended phase space is denoted by $u(u_\Delta, \eta, \mathcal{P})$. Using (8a), the generator of the BRST transformation for Abelian theories can be written as (Henneaux, 1985)

$$\Omega = \int d^2x(C\Lambda^1 - i\mathcal{P}\Lambda^0) = \int d^2x \left\{ C \left[J_0 - \left(\partial_i \pi^i + \frac{\kappa}{4} \varepsilon^{ij} F_{ij} \right) \right] - i\mathcal{P}\pi^0 \right\} \tag{9}$$

From (9), one obtains the following BRST transformation of the system

$$\begin{array}{ll} \delta A_i = \{A_i, \Omega\} = \partial_i C & \delta A_0 = \{A_0, \Omega\} = -i\mathcal{P} \\ \delta\pi^i = \{\pi^i, \Omega\} = 0 & \delta\pi^0 = \{\pi^0, \Omega\} = 0 \\ \delta\varphi = \{\varphi, \Omega\} = iC\varphi & \delta\varphi^* = \{\varphi^*, \Omega\} = -iC\varphi^* \\ \delta\pi_\varphi = \{\pi_\varphi, \Omega\} = -iC\pi_\varphi & \delta\pi^*_\varphi = \{\pi^*_\varphi, \Omega\} = iC\pi^*_\varphi \\ \delta C = \{C, \Omega\} = 0 & \delta\mathcal{P} = \{\mathcal{P}, \Omega\} = 0 \\ \delta\overline{\mathcal{P}} = \{\overline{\mathcal{P}}, \Omega\} = -J_0 + (\partial_i\pi^i + \frac{\kappa}{4}\varepsilon^{ij}F_{ij}), & \delta\overline{C} = \{\overline{C}, \Omega\} = i\pi^0 \end{array} \tag{10}$$

In order to obtain the BRST invariant Hamiltonian H in the extended phase space u, we have to calculate

$$\{\mathcal{H}_0(x), G_a(x')\} = V_a^b G_b \approx 0 \tag{11}$$

$$\{G_a(x), G_b(x')\} = C_{ab}^c G_c \approx 0 \tag{12}$$

From (11) and (12), one obtains (Henneaux, 1985)

$$H = \int d^2x\mathcal{H}_0 \tag{13}$$

The effective Hamiltonian H_{eff} in the BRST scheme is given by

$$H_{\text{eff}} = H - \{\psi, \Omega\} = \int d^2x\mathcal{H}_{\text{eff}} \tag{14}$$

Choosing ψ as $\psi = \int d^2x(i\overline{C}\chi + \overline{\mathcal{P}}\lambda)$ with $\chi = \partial_i A^i$, from (9) one obtains (Henneaux, 1985)

$$\begin{aligned} \{\psi, \Omega\} &= \int d^2x(-\lambda_1\Lambda^1 - \pi\chi + i\overline{C}\{\chi, \Lambda^1\}C - i\overline{\mathcal{P}}\mathcal{P}) \\ &= \int d^2x\{-\lambda_1\Lambda^1 - \pi^1\partial_i A^i - i\overline{C}\partial_i\partial^i C - i\overline{\mathcal{P}}\mathcal{P}\} \end{aligned} \tag{15}$$

Substituting (15) into (14), the effective action can be written as

$$\begin{aligned} I_{\text{eff}} &= \int d^3x(\pi^k\dot{A}_k + \pi_\varphi\dot{\varphi} + \pi^*_\varphi\dot{\varphi}^* + \pi_1\dot{\lambda}^1 + \dot{C}\overline{\mathcal{P}} + \dot{\mathcal{P}}\overline{C} - \mathcal{H}_{\text{eff}}) \\ &= \int d^3x(\pi^k\dot{A}_k + \pi_\varphi\dot{\varphi} + \pi^*_\varphi\dot{\varphi}^* - \mathcal{H}_0 + \dot{C}\overline{\mathcal{P}} - \dot{\overline{C}}\mathcal{P} - i\overline{\mathcal{P}}\mathcal{P} \\ &\quad - i\overline{C}\partial_i\partial^i C - \lambda_1\Lambda^1 - \pi^1\partial_\mu A^\mu) \end{aligned} \tag{16}$$

The path integral of the system in BRST formulation is given by (Henneaux, 1985)

$$\mathrm{Z}[0] = \int \mathcal{D}A_i \mathcal{D}\pi^i \mathcal{D}\lambda_1 \mathcal{D}\pi^1 \mathcal{D}\varphi \mathcal{D}\pi_\varphi \mathcal{D}\varphi^* \mathcal{D}\pi_\varphi^* \mathcal{D}C \mathcal{D}\bar{\mathcal{P}} \mathcal{D}\bar{C} \mathcal{D}\mathcal{P}$$

$$\times \exp\left\{ i \int d^3x [\pi^k \dot{A}_k + \pi_\varphi \dot{\varphi} + \pi_\varphi^* \dot{\varphi}^* - \mathcal{H}_0 + \dot{C}\bar{\mathcal{P}} - \dot{\bar{C}}\mathcal{P} - i\bar{\mathcal{P}}\mathcal{P} - i\bar{C}\partial_i \partial^i C - \lambda_1 \Lambda^1 - \pi^1 \partial_\mu A^\mu] \right\} \tag{17a}$$

Using the following results (Garcia and Vergara, 1996)

$$\int \mathcal{D}\bar{\mathcal{P}} \mathcal{D}\mathcal{P} \mathcal{D}\bar{C} \mathcal{D}C \exp\left\{ i \int_{t_1}^{t_2} dt(-\mathcal{P}\dot{\bar{C}} + \dot{C}\bar{\mathcal{P}} - i\bar{\mathcal{P}}\mathcal{P}) \right\}$$

$$= \int \mathcal{D}\bar{C} \mathcal{D}C \exp\left\{ \int_{t_1}^{t_2} dt \dot{\bar{C}}\dot{C} \right\} = -(t_2 - t_1)$$

and integrating over the ghost fields C, $\bar{\mathcal{P}}$, $\bar{C}$, and $\mathcal{P}$, one obtains

$$Z[0] = \int \mathcal{D}A_i \mathcal{D}\pi^i \mathcal{D}\lambda_1 \mathcal{D}\pi^1 \mathcal{D}\varphi \mathcal{D}\pi_\varphi \mathcal{D}\varphi^* \mathcal{D}\pi_\varphi^*$$

$$\times \exp\left\{ i \int d^3x [\pi^k \dot{A}_k + \pi_\varphi \dot{\varphi} + \pi_\varphi^* \dot{\varphi}^* - \mathcal{H}_0 - \lambda_1 \Lambda^1 - \pi^1 \partial_\mu A^\mu] \right\} \tag{17b}$$

Integrating over the Lagrange multiplier field $\lambda_1 = A_0$ and $\pi_1 = \pi_0$ in (17b), one gets

$$Z[0] = \int \mathcal{D}A_k \mathcal{D}\pi^\kappa \mathcal{D}\varphi \mathcal{D}\pi_\varphi \mathcal{D}\varphi^* \mathcal{D}\pi_\varphi^* \delta(\Lambda^1) \delta(\partial_i A^i)$$

$$\times \exp\left\{ i \int d^3x (\pi^k \dot{A}_k + \pi_\varphi \dot{\varphi} + \pi_\varphi^* \dot{\varphi}^* - \mathcal{H}_0) \right\} \tag{17c}$$

The result (17c) agrees with the conclusion obtained by using Faddeev–Senjanovic path-integral quantization scheme (Jiang and Li, 1999). The generating functional of Green functions for this system is given by (Senjanovic, 1976)

$$Z[J^\mu, J, J^*] = \int \mathcal{D}A_\mu \mathcal{D}\pi^\mu \mathcal{D}\varphi \mathcal{D}\pi_\varphi \mathcal{D}\varphi^* \mathcal{D}\pi_\varphi^*$$

$$\times \exp\left\{ i I_{\text{eff}}^P + i \int d^3x [J^\mu A_\mu + J\varphi + J^* \varphi^*] \right\} \tag{18}$$

where $I_{\text{eff}}^P = \int d^3x [\pi^\mu \dot{A}_\mu + \pi_\varphi \dot{\varphi} + \pi_\varphi^* \dot{\varphi}^* - \mathcal{H}_0 - A_0 \Lambda^1 - \pi^0 \partial_i A^i]$. We denote $\phi = (A_\mu, \varphi, \varphi^*)$, $\pi = (\pi^\mu, \pi_\varphi, \pi_\varphi^*)$, and $J = (J^\mu, J, J^*)$, thus, the generating functional (18) can be rewritten as

$$Z[J] = \int \mathcal{D}\phi \mathcal{D}\pi \exp\left[i I_{\text{eff}}^P + i \int d^3x J\phi \right] \tag{19}$$

3. FRACTIONAL SPIN AND STATISTICS

To study fractional spin, we first derive the angular momentum at the quantum level. If the effective action I^P_{eff} in (18) is invariant under the transformation

$$\begin{cases} x^{\mu'} = x^\mu + \Delta x^\mu = x^\mu + \varepsilon_\sigma \tau^{\mu\sigma}(x;\phi,\pi) \\ \phi'(x') = \phi(x) + \Delta\phi(x) = \phi(x) + \varepsilon_\sigma \xi^\sigma(x;\phi,\pi) \\ \pi'(x') = \pi(x) + \Delta\pi(x) = \pi(x) + \varepsilon_\sigma \eta^\sigma(x;\phi,\pi) \end{cases} \tag{20}$$

where $\varepsilon_\sigma(\sigma = 1, 2, \ldots, r)$ are infinitesimal parameters, and the Jacobian of corresponding transformation is equal to unity, then one can obtain the following canonical Noether theorem in quantum formalism (Li, 1996)

$$Q^\sigma = \int_V d^3x\left[\pi\left(\xi^\sigma - \phi_{,k}\tau^{k\sigma}\right) - \mathcal{H}_{\text{eff}}\tau^{0\sigma}\right] = const \tag{21}$$

Obviously, the effective canonical action I^p_{eff} in (18) is invariant under the spatial rotation transformation in (x_1, x_2) plan, and the Jacobian of the transformation of the vector $A^\alpha_\mu(x)$, scalar field $\varphi(x)$, and $\varphi^*(x)$ and their canonical momenta under the spatial rotation are equal to unity, and $\tau^{0\sigma} = 0$ in the spatial rotation. According to (21), we obtain the quantal conserved angular momentum for this system:

$$L = \int d^2x\varepsilon^{ij}\left[\pi_k S^{kl}_{ij} A_l + x_i\pi^k\partial_j A_k + x_i(\pi_\varphi\partial_j\varphi + \pi^*_\varphi\partial_j\varphi^*)\right] \tag{22}$$

where $S^{ij}_{kl} = \delta^i_k\delta^j_l - \delta^i_l\delta^j_k$. This quantum conserved angular momentum under the rotation in (x_1, x_2) plan coincides with the result derived from classical Noether theorem. In the following we show that how the fractional spin appears. Substituting (2a) into (22), one gets

$$\begin{aligned} L = &\int d^2x\left[\varepsilon^{ij}F_{k0}S^{kl}_{ij}A_l + \varepsilon^{ij}x_iF^{ko}\partial_jA_k + \varepsilon^{ij}x_i(\pi_\varphi\partial_j\varphi + \pi^*_\varphi\partial_j\varphi^*)\right] \\ &+ \int d^2x\left[\varepsilon^{ij}\frac{\kappa}{2}\varepsilon_{kj'}A^{j'}\left(S^{kl}_{ij}A_l + x_i\partial_jA^k\right)\right] \end{aligned} \tag{23}$$

Using the relations $\varepsilon^{jk}\varepsilon_{il} = \delta^j_i\delta^k_l - \delta^j_l\delta^k_i$ (Banerjee and Chakraborty, 1994) Eq. (23) is simplified to

$$\begin{aligned} L = &\int d^2x\left[\varepsilon^{ij}F_{k0}S^{kl}_{ij}A_l + \varepsilon^{ij}x_iF^{k0}\partial_jA_k + \varepsilon^{ij}x_i(\pi_\varphi\partial_j\varphi + \pi^*_\varphi\partial_j\varphi^*)\right] \\ &-\frac{\kappa}{2}\int d^2x\varepsilon^{ij}x_iA_j(\varepsilon^{lk}\partial_lA_k) \end{aligned} \tag{24}$$

We will see that the second part of the right hand in Eq. (24) implies existence of the fractional spin for this system.

Since the system is invariant under the BRST transformation, it is required that the physical state must also be invariant under such transformation. This requirement may be represented as

$$Q_{\text{BRST}}|phys\rangle = 0 \tag{25a}$$

which is the condition that the physical state must be satisfied. The BRST charge (9) can be written as

$$\Omega_{\text{BRST}} = \int d^2x \left\{ C\left[J_0 - \left(\partial_i \pi^i + \frac{\kappa}{4}\varepsilon^{ij}F_{ij} \right) \right] \right\} + \int d^2x(-i\mathcal{P}\pi^0)$$

The physical state condition (25a) can be reduced to (Kim *et al.*, 1994)

$$\int d^2x \left[J_0 - \left(\partial_i \pi^i + \frac{\kappa}{4}\varepsilon^{ij}F_{ij} \right) \right] |phys\rangle \otimes (C|0\rangle_{\text{gh}}) = 0 \tag{25b}$$

where $|0\rangle_{\text{gh}}$ is the ghost vacuum state. By the independence of C with other fields, we obtain

$$[J_0 - \nabla^2 A^0 - \partial_i \dot{A}^i - \kappa\varepsilon^{ij}\partial_i A_j]|phys\rangle = 0 \tag{25c}$$

According the gauge condition $\partial_i A^i \approx 0$, one has

$$\partial_i \dot{A}^i|phys\rangle = 0 \tag{26a}$$

i.e.

$$\left[\partial_i \pi^i + \nabla^2 A^0 - \frac{\kappa}{2}\varepsilon^{ij}\partial_i A_j \right] |phys\rangle = 0 \tag{26b}$$

From (25c) and (26), one gets

$$[J_0 - \kappa\varepsilon^{ij}\partial_i A_j]|phys\rangle = 0 \tag{27}$$

If we solve Eq. (27), we obtain (Kim *et al.*, 1994)

$$A_i(x) = -\frac{\pi}{\kappa}\varepsilon_{ij}\partial_x^j \int d^2y G(x-y)J_0(y) \tag{28}$$

From (24), (27), and (28), one can obtain (Kim *et al.*, 1994)

$$\begin{aligned} L &= \int d^2x \left[\varepsilon^{ij}F_{k0}S_{ij}^{kl}A_l + \varepsilon^{ij}x_i(F^{k0}\partial_j A_k + \pi_\varphi \partial_j \varphi + \pi_\varphi^* \partial_j \varphi^*) \right] \\ &\quad - \frac{1}{2}\int d^2x \varepsilon^{ij}x_j A_i J_0 \\ &= \int d^2x \left[\varepsilon^{ij}F_{k0}S_{ij}^{kl}A_l + \varepsilon^{ij}x_i(F^{i'0}\partial_j A_{i'} + \pi_\varphi \partial_j \varphi + \pi_\varphi^* \partial_j \varphi^*) \right] + \frac{Q^2}{4\kappa} \end{aligned} \tag{29}$$

where $Q = \int d^2x J_0$, the first term on the right-hand side of Eq. (29) is the canonical angular momentum operator and the second is the anomalous one which is interpreted as a spin operator (Kim *et al.*, 1994). Therefore, anyons still survive in the Maxwell CS theories with relativistic matter. However, the fractional spin $\frac{Q^2}{4\kappa}$ of the system is different from its value without the Maxwell kinetic term (Kim *et al.*, 1994).

4. CANONICAL WARD IDENTITIES

Let us now construct the gauge transformation for a system with Lagrangian (1). Dirac in his work on the generalized canonical formalism conjectured that all first-class constraints are generators of the gauge transformation. In spite of the lack of a proof of this conjecture we do not know of any physically important system for which Dirac's conjecture leads to the wrong result. (Li, 1991) has shown that for a system with both primary first-class and secondary first-class constraints the generator of gauge transformation can be constructed by using all first-class constraints. For a system with Lagragian (1), the first-class constraints are (3) and (6). The gauge generator for this system can be written as (Li, 1991)

$$\begin{aligned} G &= \int d^2y[\varepsilon(y)\Lambda_1 - \varepsilon(y)_{,0}\Lambda_0] \\ &= \int d^2y\left[i(\pi_\varphi\varphi - \varphi^*\pi_\varphi^*)\varepsilon(y) - \frac{\kappa}{4}\varepsilon^{ij}F_{ij}\varepsilon(y) + \pi^\mu\partial_\mu\varepsilon(y)\right] \end{aligned} \tag{30}$$

This generator produces the following transformation:

$$\begin{cases} \delta\varphi = \{\varphi(x), G\} = i\varphi(x)\varepsilon(x), & \delta\pi_\varphi = \{\pi_\varphi(x), G\} = -i\pi_\varphi(x)\varepsilon(x) \\ \delta\varphi^* = \{\varphi^*(x), G\} = -i\varphi^*(x)\varepsilon(x), & \delta\pi^* = \{\pi_\varphi^*(x), G\} = i\pi_\varphi^*(x)\varepsilon(x) \\ \delta A_\mu = \{A_\mu(x), G\} = \partial_\mu\varepsilon(x), & \delta\pi^\mu = \{\pi^\mu(x), G\} = \frac{\kappa}{2}\varepsilon^{\mu i}\partial_i\varepsilon(x) \end{cases} \tag{31}$$

Under this transformation the Lagrangian is changed only by a divergence term, hence the canonical action is invariant under (31). The Jacobian of the transformation (31) is equal to unity. Thus the generating functional (18) is invariant under the transformation (31); this yields the following Ward identity (Li, 1996):

$$\left\{\partial^\mu\partial_\mu\pi^0 - \partial^\mu J_\mu - J\frac{\delta}{\delta J} + J^*\frac{\delta}{\delta J^*}\right\}Z[J] = 0 \tag{32}$$

Let $Z[J] = \exp\{iW[J]\}$ and use the definition of the generating functional of proper vertices $\Gamma[\phi]$ which is given by performing a functional Legendre transformation on $W[J]$

$$\Gamma[\phi] = W[J] - \int d^3x J\phi \tag{33}$$

$$\frac{\delta W}{\delta J(x)} = \phi(x), \qquad \frac{\delta\Gamma}{\delta\phi(x)} = -J(x) \tag{34}$$

Then the Ward identity (32) becomes

$$\partial_\mu \partial^\mu \pi^0 - \partial^\mu \frac{\delta \Gamma}{\delta A^\mu} - \frac{\delta \Gamma}{\delta \varphi}\varphi + \frac{\delta \Gamma}{\delta \varphi^*}\varphi^* = 0 \tag{35}$$

Functionally differentiating (35) with respect to $\varphi(x_2)$ and $\varphi(x_3)$, and setting all fields equal to zero, we obtain

$$\partial^\mu_{x_1} \frac{\delta^3 \Gamma(0)}{\delta A^\mu(x_1)\delta\varphi(x_2)\delta\varphi(x_3)} - \frac{\delta^2 \Gamma(0)}{\delta\varphi(x_1)\delta\varphi(x_2)}\delta(x_1 - x_3) - \frac{\delta^3 \Gamma(0)}{\delta\varphi(x_1)\delta\varphi(x_3)}\delta(x_1 - x_2) = 0 \tag{36}$$

Similarly, differentiating (35) many times with respect to field variables and setting all fields equal to zero, one can obtain various Ward identities for proper vertices.

5. CONCLUSION

In the BRST path integral framework, we have quantized the CS theory with Maxwell term. On the basis of the symmetries of constrained canonical (Hamiltonian) systems, we derived a generalized spin-statistics relation at the quantum level by computing the angular momentum though the quantum Noether law. However, the value of the fractional spin of the system is different the value without the Maxwell kinetic term. Finally, the gauge generator is constructed, and the associated Ward identities for this system are given, from which some relationships of the proper vertices can be obtained. This formulation for deriving Ward identities of proper vertices has a significant advantage in that one does not need to carry out explicit integration over canonical momenta as one usually did.

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